

CMPTG 5 - HW 5 - MLP Statistics

Sidharth Kannan

April 2025

1. Argue why for any multivariate Gaussian, $\mathbf{z} \sim \mathcal{N}(0, K)$, there exists a basis in which the entries of the random vector \mathbf{z} are independent.
2. Apply Wick's theorem to write $\mathbb{E}[z_1 z_2 z_3 z_4 z_5 z_6]$ of a multivariate Gaussian $\mathbf{z} \sim \mathcal{N}(0, K)$ as a sum of products of covariances. Your answer should be a sum with 15 terms.
3. Consider the example of a *deep linear network* with the following properties. Input size = 2, 1 hidden layer with dimension 1, and output size = 2. Like in class, assume there are no biases.
 - (a) What are the dimensions of the weight matrices?
 - (b) Since this network is a product of matrices, it can be represented as a single matrix, \mathcal{W} . What would be the dimension of that matrix?
 - (c) Can this network represent all matrices of the same dimension as your answer for part (b)? If so, argue as to why. If not, give an example of a matrix, with the same dimensions as part (b), that this network cannot represent.