CMPTG 5 - HW 2 - Ising Model and Ising Machines

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Questions 1, 2, and 3 are programming questions. Question 3 is a physics problem.

1. Implement a simple Ising machine in the programming language of your choice. That is, implement the following pieces of functionality.
    (a) Implement a binary stochastic neuron that takes some input and determines its next state.
    (b) Implement a way to couple multiple binary stochastic neurons
    (c) Test it out with the simple networks we demonstrated in class!
2. Now, take your Ising machine and setup a 4 x 4 square lattice, like that shown in the figure. Implement the simulated annealing algorithm to find the ground state of the lattice. Check that you have achieved the correct ground state by comparing against a brute force approach.

    To implement simulated annealing, after each pass over the whole network, reduce the temperature by multiplying it by some constant < 1.
3. In class we showed that we cannot update spins in parallel in the simple case of two connected spins, or the network will not converge to the Boltzmann distribution. Serial sampling, however, kills the scalability of our computation.

    It turns out that there is, however, a compromise. Instead of pure serial or pure parallel, we can update disconnected spins in parallel. This is called *blocked Gibbs sampling*. Implement blocked Gibbs sampling on our 4 by 4 spin glass. Compare the performance in terms of a) wall clock time and b) number of network updates to find the ground state. For (b), run the algorithm multiple times and report the mean + standard deviation.

    Note: The square lattice graph is 2-colorable, so you can update the network in just two blocks.
4. An ferromagnetic Ising model is a lattice of binary spins, where each spin experiences some sort of attractive coupling to its neighbors.

The Ising Hamiltonian (energy) with no external field is written as follows:

$$E = -\sum_{i} \sum_{j \in \mathcal{N}(i)} J s_i s_j \tag{1}$$

where J denotes the interaction strength, s_i denotes the spin of node i, and $\mathcal{N}(i)$ is the set of all neighbors of node i.

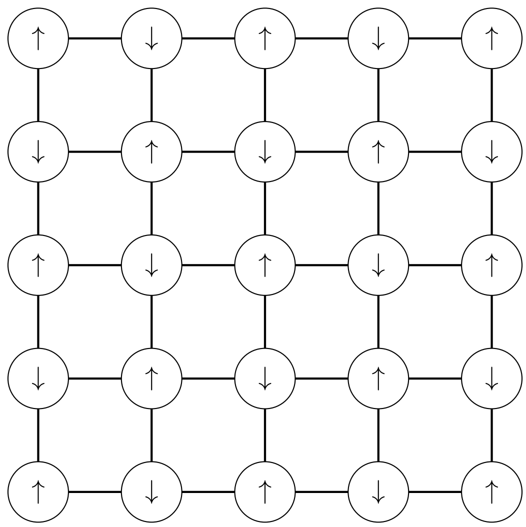

Figure 1: An Illustration of the 2D Ising lattice.

(a) Using the mean-field approximation

$$h_{eff} = J \sum_{i \in \mathcal{N}()} \langle s_i \rangle = Jzm \tag{2}$$

Derive an expression for the average magnetization of the lattice.

$$m = \langle s \rangle \tag{3}$$

(b) Derive an expression for the critical temperature, T_c.

(c) Plot the average magnetization as a function of temperature for $J = 0.1, 1, 10$.

(d) Does the same phase transition that you observed happen in 1D?